Differential Equations Springer

Fractional calculus was first developed by pure mathematicians in the middle of the 19th century. Some 100 years later, engineers and physicists have found applications for these concepts in their areas. However there has traditionally been little interaction between these two communities. In particular, typical mathematical works provide extensive findings on aspects with comparatively little significance in applications, and the engineering literature often lacks mathematical detail and precision. This book bridges the gap between the two communities. It concentrates on the class of fractional derivatives most important in applications, the Caputo operators, and provides a self-contained, thorough and mathematically rigorous study of their properties and of the corresponding differential equations. The text is a useful tool for mathematicians and researchers from the applied sciences alike. It can also be used as a basis for teaching graduate courses on fractional differential equations.
This work gathers a selection of outstanding papers presented at the 25th Conference on Differential Equations and Applications / 15th Conference on Applied Mathematics, held in Cartagena, Spain, in June 2017. It supports further research into both ordinary and partial differential equations, numerical analysis, dynamical systems, control and optimization, trending topics in numerical linear algebra, and the applications of mathematics to industry. The book includes 14 peer-reviewed contributions and mainly addresses researchers interested in the applications of mathematics, especially in science and engineering. It will also greatly benefit PhD students in applied mathematics, engineering and physics.
This book presents a variety of techniques for solving ordinary differential equations analytically and features a wealth of examples. Focusing on the modeling of real-world phenomena, it begins with a basic introduction to differential equations, followed by linear and nonlinear first order equations and a detailed treatment of the second order linear equations. After presenting solution methods for the Laplace transform and power series, it lastly presents systems of equations and offers an introduction to the stability theory.To help readers practice the theory covered, two types of exercises are provided: those that illustrate the general theory, and others designed to expand on the text material. Detailed solutions to all the exercises are included.The book is excellently suited for use as a textbook for an undergraduate class (of all disciplines) in ordinary differential equations.
The book serves both as a reference for various scaled models with corresponding dimensionless numbers, and as a resource for learning the art of scaling. A special feature of the book is the emphasis on how to create software for scaled models, based on existing software for unscaled models. Scaling (or non-dimensionalization) is a mathematical technique that greatly simplifies the setting of input parameters in numerical simulations. Moreover, scaling enhances the understanding of how different physical processes interact in a differential equation model. Compared to the existing literature, where the topic of scaling is frequently encountered, but very often in only a brief and shallow setting, the present book gives much more thorough explanations of how to reason about finding the right scales. This process is highly problem dependent, and therefore the book features a lot of worked examples, from very simple ODEs to systems of PDEs, especially from fluid mechanics. The text is easily accessible and example-driven. The first part on ODEs fits even a lower undergraduate level, while the most advanced multiphysics fluid mechanics examples target the graduate level. The scientific literature is full of scaled models, but in most of the cases, the scales are just stated without thorough mathematical reasoning. This book explains how the scales are found mathematically. This book will be a valuable read for anyone doing numerical simulations based on ordinary or partial differential equations.
This book aims to establish a foundation for fractional derivatives and fractional differential equations. The theory of fractional derivatives enables considering any positive order of differentiation. The history of research in this field is very long, with its origins dating back to Leibniz. Since then, many great mathematicians, such as Abel, have made contributions that cover not only theoretical aspects but also physical applications of fractional calculus. The fractional partial differential equations govern phenomena depending both on spatial and time variables and require more subtle treatments. Moreover, fractional partial differential equations are highly demanded model equations for solving real-world problems such as the anomalous diffusion in heterogeneous media. The studies of fractional partial differential equations have continued to expand explosively. However we observe that available mathematical theory for fractional partial differential equations is not still complete. In particular, operator-theoretical approaches are indispensable for some generalized categories of solutions such as weak solutions, but feasible operator-theoretic foundations for wide applications are not available in monographs. To make this monograph more readable, we are restricting it to a few fundamental types of time-fractional partial differential equations, forgoing many other important and exciting topics such as stability for nonlinear problems. However, we believe that this book works well as an introduction to mathematical research in such vast fields.
The book is a primer of the theory of Ordinary Differential Equations. Each chapter is completed by a broad set of exercises; the reader will also find a set of solutions of selected exercises. The book contains many interesting examples as well (like the equations for the electric circuits, the pendulum equation, the logistic equation, the Lotka-Volterra system, and many other) which introduce the reader to some interesting aspects of the theory and its applications. The work is mainly addressed to students of Mathematics, Physics, Engineering, Statistics, Computer Sciences, with knowledge of Calculus and Linear Algebra, and contains more advanced topics for further developments, such as Laplace transform; Stability theory and existence of solutions to Boundary Value problems. A complete Solutions Manual, containing solutions to all the exercises published in the book, is available. Instructors who wish to adopt the book may request the manual by writing directly to one of the authors.
The present book builds upon an earlier work of J. Hale, "Theory of Func tional Differential Equations" published in 1977. We have tried to maintain the spirit of that book and have retained approximately one-third of the material intact. One major change was a complete new presentation of lin ear systems (Chapters 6~9) for retarded and neutral functional differential equations. The theory of dissipative systems (Chapter 4) and global at tractors was completely revamped as well as the invariant manifold theory (Chapter 10) near equilibrium points and periodic orbits. A more complete theory of neutral equations is presented (see Chapters 1, 2, 3, 9, and 10). Chapter 12 is completely new and contains a guide to active topics of re search. In the sections on supplementary remarks, we have included many references to recent literature, but, of course, not nearly all, because the subject is so extensive. Jack K. Hale Sjoerd M. Verduyn Lunel Contents
Preface.. v Introduction . . . . 1 . . . . 1. Linear differential difference equations . . . . 11 . . . . . . 1.1 Differential and difference equations. 11 . . . . 1.2 Retarded differential difference equations. . . . . 13 . . . . 1.3 Exponential estimates of x(¢,f) . . . . 15 . . . . 1.4 The characteristic equation . . . . 17 . . . . 1.5 The fundamental solution. . . . . 18 . . . . 1.6 The variation-of-constants formula............................. 23 1. 7 Neutral differential difference equations . . . . 25 . . . . 1.8 Supplementary remarks. . . . . 34 . . . . 2. Functional differential equations: Basic theory . . . . 38 . . 2.1 Definition of a retarded equation. . . . . 38 . . . . 2.2 Existence, uniqueness, and continuous dependence . . . . 39 . . . 2.3 Continuation of solutions . . . . 44 . . . .
For over 300 years, differential equations have served as an essential tool for describing and analyzing problems in many scientific disciplines. This carefully-written textbook provides an introduction to many of the important topics associated with ordinary differential equations. Unlike most textbooks on the subject, this text includes nonstandard topics such as perturbation methods and differential equations and Mathematica. In addition to the nonstandard topics, this text also contains contemporary material in the area as well as its

classical topics. This second edition is updated to be compatible with Mathematica, version 7.0. It also provides 81 additional exercises, a new section in Chapter 1 on the generalized logistic equation, an additional theorem in Chapter 2 concerning fundamental matrices, and many more other enhancements to the first edition. This book can be used either for a second course in ordinary differential equations or as an introductory course for well-prepared students. The prerequisites for this book are three semesters of calculus and a course in linear algebra, although the needed concepts from linear algebra are introduced along with examples in the book. An undergraduate course in analysis is needed for the more theoretical subjects covered in the final two chapters.

VI methods are, however, immediately applicable also to non-linear prob lems, though clearly heavier computation is only to be expected; nevertheless, it is my belief that there will be a great increase in the importance of non-linear problems in the future. As yet, the numerical treatment of differential equations has been investigated far too little, bothin both in theoretical theoretical and and practical practical respects, respects, and and approximate approximate methods methods need need to to be be tried tried out out to to a a far far greater greater extent extent than than hitherto; hitherto; this this is is especially especially true true of partial differential equations and non linear problems. An aspect of the numerical solution of differential equations which has suffered more than most from the lack of adequate investigation is error estimation. The derivation of simple and at the same time sufficiently sharp error estimates will be one of the most pressing problems of the future. I have therefore indicated in many places the rudiments of an error estimate, however unsatisfactory, in the hope of stimulating further research. Indeed, in this respect the book can only be regarded as an introduction. Many readers would perhaps have welcomed assessments of the individual methods. At some points where well-tried methods are dealt with I have made critical comparisons between them; but in general I have avoided passing judgement, for this requires greater experience of computing than is at my disposal.

Numerical Methods for Ordinary Differential Equations is a self-contained introduction to a fundamental field of numerical analysis and scientific computation. Written for undergraduate students with a mathematical background, this book focuses on the analysis of numerical methods without losing sight of the practical nature of the subject. It covers the topics traditionally treated in a first course, but also highlights new and emerging themes. Chapters are broken down into `lecture' sized pieces, motivated and illustrated by numerous theoretical and computational examples. Over 200 exercises are provided and these are starred according to their degree of difficulty. Solutions to all exercises are available to authorized instructors. The book covers key foundation topics: o Taylor series methods o Runge--Kutta methods o Linear multistep methods o Convergence o Stability and a range of modern themes: o Adaptive stepsize selection o Long term dynamics o Modified equations o Geometric integration o Stochastic differential equations The prerequisite of a basic university-level calculus class is assumed, although appropriate background results are also summarized in appendices. A dedicated website for the book containing extra information can be found via www.springer.com

This is the practical introduction to the analytical approach taken in Volume 2. Based upon courses in partial differential equations over the last two decades, the text covers the classic canonical equations, with the method of separation of variables introduced at an early stage. The characteristic method for first order equations acts as an introduction to the classification of second order quasi-linear problems by characteristics. Attention then moves to different co-ordinate systems, primarily those with cylindrical or spherical symmetry. Hence a discussion of special functions arises quite naturally, and in each case the major properties are derived. The next section deals with the use of integral transforms and extensive methods for inverting them, and concludes with links to the use of Fourier series.

Few books on Ordinary Differential Equations (ODEs) have the elegant geometric insight of this one, which puts emphasis on the qualitative and geometric properties of ODEs and their solutions, rather than on routine presentation of algorithms. From the reviews: "Professor Arnold has expanded his classic book to include new material on exponential growth, predator-prey, the pendulum, impulse response, symmetry groups and group actions, perturbation and bifurcation." --SIAM REVIEW

For lecture courses that cover the classical theory of nonlinear differential equations associated with Poincare and Lyapunov and introduce the student to the ideas of bifurcation theory and chaos, this text is ideal. Its excellent pedagogical style typically consists of an insightful overview followed by theorems, illustrative examples, and exercises.

This book is based on a course I have given five times at the University of Michigan, beginning in 1973. The aim is to present an introduction to a sampling of ideas, phenomena, and methods from the subject of partial differential equations that can be presented in one semester and requires no previous knowledge of differential equations. The problems, with hints and discussion, form an important and integral part of the course. In our department, students with a variety of specialties-notably differen? tial geometry, numerical analysis, mathematical physics, complex analysis, physics, and partial differential equations-have a need for such a course. The goal of a one-term course forces the omission of many topics. Everyone, including me, can find fault with the selections that I have made. One of the things that makes partial differential equations difficult to learn is that it uses a wide variety of tools. In a short course, there is no time for the leisurely development of background material. Consequently, I suppose that the reader is trained in advanced calculus, real analysis, the rudiments of complex analysis, and the language offunctional analysis. Such a background is not unusual for the students mentioned above. Students missing one of the "essentials" can usually catch up simultaneously. A more difficult problem is what to do about the Theory of Distributions

This book provides a thorough introduction to the mathematical and algorithmic aspects of certified reduced basis methods for parametrized partial differential equations. Central aspects ranging from model construction, error estimation and computational efficiency to empirical interpolation methods are discussed in detail for coercive problems. More advanced aspects associated with time-dependent problems, non-compliant and non-coercive problems and applications with geometric variation are also discussed as examples.

Due to the fundamental role of differential equations in science and engineering it has long been a basic task of numerical analysts to generate numerical values of solutions to differential equations. Nearly all approaches to this task involve a "finitization" of the original differential equation problem, usually by a projection into a finite-dimensional space. By far the most popular of these finitization processes consists of a reduction to a difference equation problem for functions which take values only on a grid of argument points. Although some of these finite difference methods have been known for a long time, their wide applica bility and great efficiency came to light only with the spread of electronic computers. This in tum strongly stimulated research on the properties and practical use of finite-difference methods. While the theory or partial differential equations and their discrete analogues is a very hard subject, and progress is consequently slow, the initial value problem for a system of first order ordinary differential equations lends itself so naturally to discretization that hundreds of numerical analysts have felt inspired to invent an ever-increasing number of finite-difference methods for its solution. For about 15 years, there has hardly been an issue of a numerical journal without new results of this kind; but clearly the vast majority of these methods have just been variations of a few basic themes. In this situation, the classical text book by P.

The aim of this book is to give a systematic study of questions con cerning existence, uniqueness and regularity of solutions of linear partial differential equations and boundary problems. Let us note explicitly that this program does not contain such topics as eigenfunction expan sions, although we do give the main facts concerning differential operators which are required for their study. The restriction to linear equations also means that the trouble of achieving minimal assumptions concerning the smoothness of the coefficients of the differential equations studied would not be worth while; we usually assume that they are infinitely differenti able. Functional analysis and distribution theory form the framework for the theory developed here. However, only classical results of functional analysis are used. The terminology employed is that of BOURBAKI. To make the exposition self-contained we present in Chapter I the elements of distribution theory that are required. With the possible exception of section 1.8, this introductory chapter should be bypassed by a reader who is already familiar with distribution theory.

Used in undergraduate classrooms across the USA, this is a clearly written, rigorous introduction to differential equations and their applications. Fully understandable to students who have had one year of calculus, this book distinguishes itself from other differential equations texts through its engaging application of the subject matter to interesting scenarios. This fourth edition incorporates earlier introductory material on bifurcation theory and adds a new chapter on Sturm-Liouville boundary value problems. Computer programs in C, Pascal, and Fortran are presented throughout the text to show readers how to apply differential equations towards quantitative problems.

The proposed book is one of a series called "A Course of Higher Mathematics and Mathematical Physics" edited by A. N. Tikhonov, V. A. Ilyin and A. G. Sveshnikov. The book is based on a lecture course which, for a number of years now has been taught at the Physics Department and the Department of Computational Mathematics and Cybernetics of Moscow State University. The exposition reflects the present state of the theory of differential equations, as far as it is required by future specialists in physics and applied mathematics, and is at the same time elementary enough. An important part of the book is devoted to approximation methods for the solution and study of differential equations, e.g. numerical and asymptotic methods, which at the present time play an essential role in the study of mathematical models of physical phenomena. Less attention is paid to the integration of differential equations in elementary functions than to the study of algorithms on which numerical solution methods of differential equations for computers are based.

The first of three volumes on partial differential equations, this one introduces basic examples arising in continuum mechanics, electromagnetism, complex analysis and other areas, and develops a number of tools for their solution, in particular Fourier analysis, distribution theory, and Sobolev spaces. These tools are then applied to the treatment of basic problems in linear PDE, including the Laplace equation, heat equation, and wave equation, as well as more general elliptic, parabolic, and hyperbolic equations.The book is targeted at graduate students in mathematics and at professional mathematicians with an interest in partial differential equations, mathematical physics, differential geometry, harmonic analysis, and complex analysis.

Therearemanyexcellenttextsonelementarydi?erentialequationsdesignedfor the standard sophomore course. However, in spite of the fact that most courses are one semester in length, the texts have evolved into calculus-like pres- tations that include a large collection of methods and applications, packaged with student manuals, and Web-based notes, projects, and supplements. All of this comes in several hundred pages of text with busy formats. Most students do not have the time or desire to read voluminous texts and explore internet supplements. The format of this di?erential equations book is di?erent; it is a one-semester, brief treatment of the basic ideas, models, and solution methods. Itslimitedcoverageplacesitsomewherebetweenanoutlineandadetailedte- book. I have tried to write concisely, to the point, and in plain language. Many worked examples and exercises are included. A student who works through this primer will have the tools to go to the next level in applying di?erential eq- tions to problems in engineering, science, and applied mathematics. It can give some instructors, who want more concise coverage, an alternative to existing texts.

In this undergraduate/graduate textbook, the authors introduce ODEs and PDEs through 50 class-tested lectures. Mathematical concepts are explained with clarity and rigor, using fully worked-out examples and helpful illustrations. Exercises are provided at the end of each chapter for practice. The treatment of ODEs is developed in conjunction with PDEs and is aimed mainly towards applications. The book covers important applications-oriented topics such as solutions of ODEs in form of power series, special functions, Bessel functions, hypergeometric functions, orthogonal functions and polynomials, Legendre, Chebyshev, Hermite, and Laguerre polynomials, theory of Fourier series. Undergraduate and graduate students in mathematics, physics and engineering will benefit from this book. The book assumes familiarity with calculus.

The subject of this book is the solution of stiff differential equations and of differential-algebraic systems. This second edition contains new material including new numerical tests, recent progress in numerical differential-algebraic equations, and improved FORTRAN codes. From the reviews: "A superb book...Throughout, illuminating graphics, sketches and quotes from papers of researchers in the field add an element of easy informality and motivate the text." --MATHEMATICS TODAY

This book offers an ideal graduate-level introduction to the theory of partial differential equations. The first part of the book describes the basic mathematical problems and structures associated with elliptic, parabolic, and hyperbolic partial differential equations, and explores the connections between these fundamental types. Aspects of Brownian motion or pattern formation processes are also presented. The second part focuses on existence schemes and develops estimates for solutions of elliptic equations, such as Sobolev space theory, weak and strong solutions, Schauder estimates, and Moser iteration. In particular, the reader will learn the basic techniques underlying current research in elliptic partial differential equations. This revised and expanded third edition is enhanced with many additional examples that will help motivate the reader. New features include a reorganized and extended chapter on hyperbolic equations, as well as a new chapter on the relations between different types of partial differential equations, including first-order hyperbolic systems, Langevin and Fokker-Planck equations, viscosity solutions for elliptic PDEs, and much more. Also, the new edition contains additional material on systems of elliptic partial differential equations, and it explains in more detail how the Harnack inequality can be used for the regularity of solutions.

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Author received the 1962 Fields Medal Author received the 1988 Wolf Prize (honoring achievemnets of a lifetime) Author is leading expert in partial differential equations

This textbook is a comprehensive treatment of ordinary differential equations, concisely presenting basic and essential results in a rigorous manner. Including various examples from physics, mechanics, natural sciences, engineering and automatic theory, Differential Equations is a bridge between the abstract theory of differential equations and applied systems theory. Particular attention is given to the existence and uniqueness of the Cauchy problem, linear differential systems, stability theory and applications to first-order partial differential equations. Upper undergraduate students and researchers in applied mathematics and systems theory with a background in advanced calculus will find this book particularly useful. Supplementary topics are covered in an appendix enabling the book to be completely self-contained.

This book is a mathematically rigorous introduction to the beautiful subject of ordinary differential equations for beginning graduate or advanced undergraduate students. Students should have a solid background in analysis and linear algebra. The presentation emphasizes commonly used techniques without necessarily striving for completeness or for the treatment of a large number of topics. The first half of the book is devoted to the development of the basic theory: linear systems, existence and uniqueness of solutions to the initial value problem, flows, stability, and smooth dependence of solutions upon initial conditions and parameters. Much of this theory also serves as the paradigm for evolutionary partial differential equations. The second half of the book is devoted to geometric theory: topological conjugacy, invariant manifolds, existence and stability of periodic solutions, bifurcations, normal forms, and the existence of transverse homoclinic points and their link to chaotic dynamics. A common thread throughout the second part is the use of the implicit function theorem in Banach space. Chapter 5, devoted to this topic, the serves as the bridge between the two halves of the book.

This book deals with methods for solving nonstiff ordinary differential equations. The first chapter describes the historical development of the classical theory, and the second chapter includes a modern treatment of Runge-Kutta and extrapolation methods. Chapter three begins with the classical theory of multistep methods, and concludes with the theory of general linear methods. The reader will benefit from many illustrations, a historical and didactic approach, and computer programs which help him/her learn to solve all kinds of ordinary differential equations. This new edition has been rewritten and new material has been included.

1. The development of a theory of optimal control (deterministic) requires the following initial data: (i) a control u belonging to some set ilIi ad (the set of 'admissible controls') which is at our disposition, (ii) for a given control u, the state y(u) of the system which is to be controlled is given by the solution of an equation (*) Ay(u)=given function ofu where A is an operator (assumed known) which specifies the system to be controlled (A is the 'model' of the system), (iii) the observation z(u) which is a function of y(u) (assumed to be known exactly; we consider only deterministic problems in this book), (iv) the "cost function" J(u) ("economic function") which is defined in terms of a numerical function z-+

This textbook is designed for a one year course covering the fundamentals of partial differential equations, geared towards advanced undergraduates and beginning graduate students in mathematics, science, engineering, and elsewhere. The exposition carefully balances solution techniques, mathematical rigor, and significant applications, all illustrated by numerous examples. Extensive exercise sets appear at the end of almost every subsection, and include straightforward computational problems to develop and reinforce new techniques and results, details on theoretical developments and proofs, challenging projects both computational and conceptual, and supplementary material that motivates the student to delve further into the subject. No previous experience with the subject of partial differential equations or Fourier theory is assumed, the main prerequisites being undergraduate calculus, both one- and multi-variable, ordinary differential equations, and basic linear algebra. While the classical topics of separation of variables, Fourier analysis, boundary value problems, Green's functions, and special functions continue to form the core of an introductory course, the inclusion of nonlinear equations, shock wave dynamics, symmetry and similarity, the Maximum Principle, financial models, dispersion and solitons, Huygens'. Principle, quantum mechanical systems, and more make this text well attuned to recent developments and trends in this active field of contemporary research. Numerical approximation schemes are an important component of any introductory course, and the text covers the two most basic approaches: finite differences and finite elements. Peter J. Olver is professor of mathematics at the University of Minnesota. His wide-ranging research interests are centered on the development of symmetry-based methods for differential equations and their manifold applications. He is the author of over 130 papers published in major scientific research journals as well as 4 other books, including the definitive Springer graduate text, Applications of Lie Groups to Differential Equations, and another undergraduate text, Applied Linear Algebra. A Solutions Manual for instrucors is available by clicking on "Selected Solutions Manual" under the Additional Information section on the right-hand side of this page.

The book is intended as an advanced undergraduate or first-year graduate course for students from various disciplines, including applied mathematics, physics and engineering. It has evolved from courses offered on partial differential equations (PDEs) over the last several years at the Politecnico di Milano. These courses had a twofold purpose: on the one hand, to teach students to appreciate the interplay between theory and modeling in problems arising in the applied sciences, and on the other to provide them with a solid theoretical background in numerical methods, such as finite elements. Accordingly, this textbook is divided into two parts. The first part, chapters 2 to 5, is more elementary in nature and focuses on developing and studying basic problems from the macro-areas of diffusion, propagation and transport, waves and vibrations. In turn the second part, chapters 6 to 11, concentrates on the development of Hilbert spaces methods for the variational formulation and the analysis of (mainly) linear boundary and initial-boundary value problems.The third edition contains a few text and formulas revisions and new exercises.

This volume provides an introduction to the analytical and numerical aspects of partial differential equations (PDEs). It unifies an analytical and computational approach for these; the qualitative behaviour of solutions being established using classical concepts: maximum principles and energy methods. Notable inclusions are the treatment of irregularly shaped boundaries, polar coordinates and the use of flux-limiters when approximating hyperbolic conservation laws. The numerical analysis of difference schemes is rigorously developed using discrete maximum principles and discrete Fourier analysis. A novel feature is the inclusion of a chapter containing projects, intended for either individual or group study, that cover a range of topics such as parabolic smoothing, travelling waves, isospectral matrices, and the approximation of multidimensional advection–diffusion problems. The underlying theory is illustrated by numerous examples and there are around 300 exercises, designed to promote and test understanding. They are starred according to level of difficulty. Solutions to odd-numbered exercises are available to all readers while even-numbered solutions are available to authorised instructors. Written in an informal yet rigorous style, Essential Partial Differential Equations is designed for mathematics undergraduates in their final or penultimate year of university study, but will be equally useful for students following other scientific and engi neering disciplines in which PDEs are of practical importance. The only prerequisite is a familiarity with the basic concepts of calculus and linear algebra.

There are three major changes in the Third Edition of Differential Equations and Their Applications. First, we have completely rewritten the section on singular solutions of differential equations. A new section, 2.8.1, dealing with Euler equations has been added, and this section is used to motivate a greatly expanded treatment of singular equations in sections 2.8.2 and 2.8.3. Our second major change is the addition of a new section, 4.9, dealing with bifurcation theory, a subject of much current interest. We felt it desirable to give the reader a brief but nontrivial introduction to this important topic. Our third major change is in Section 2.6, where we have switched to the metric system of units. This change was requested by many of our readers. In addition to the above changes, we have updated the material on population models, and have revised the exercises in this section. Minor editorial changes have also been made throughout the text. New York City Martin Braun Nooember, 1982 Preface to the First Edition This textbook is a unique blend of the theory of differential equations and their exciting application to "real world" problems. First, and foremost, it is a rigorous study of ordinary differential equations and can be fully understood by anyone who has completed one year of calculus. However, in addition to the traditional applications, it also contains many exciting "real life" problems. These applications are

completely self contained.
This book is a very well-accepted introduction to the subject. In it, the author identifies the significant aspects of the theory and explores them with a limited amount of machinery from mathematical analysis. Now, in this fourth edition, the book has again been updated with an additional chapter on Lewy's example of a linear equation without solutions.
Taking readers with a basic knowledge of probability and real analysis to the frontiers of a very active research discipline, this textbook provides all the necessary background from functional analysis and the theory of PDEs. It covers the main types of equations (elliptic, hyperbolic and parabolic) and discusses different types of random forcing. The objective is to give the reader the necessary tools to understand the proofs of existing theorems about SPDEs (from other sources) and perhaps even to formulate and prove a few new ones. Most of the material could be covered in about 40 hours of lectures, as long as not too much time is spent on the general discussion of stochastic analysis in infinite dimensions. As the subject of SPDEs is currently making the transition from the research level to that of a graduate or even undergraduate course, the book attempts to present enough exercise material to fill potential exams and homework assignments. Exercises appear throughout and are usually directly connected to the material discussed at a particular place in the text. The questions usually ask to verify something, so that the reader already knows the answer and, if pressed for time, can move on. Accordingly, no solutions are provided, but there are often hints on how to proceed. The book will be of interest to everybody working in the area of stochastic analysis, from beginning graduate students to experts in the field.
Ordinary differential equations serve as mathematical models for many exciting real world problems. Rapid growth in the theory and applications of differential equations has resulted in a continued interest in their study by students in many disciplines. This textbook organizes material around theorems and proofs, comprising of 42 class-tested lectures that effectively convey the subject in easily manageable sections. The presentation is driven by detailed examples that illustrate how the subject works. Numerous exercise sets, with an "answers and hints" section, are included. The book further provides a background and history of the subject.
The book comprises a rigorous and self-contained treatment of initial-value problems for ordinary differential equations. It additionally develops the basics of control theory, which is a unique feature in current textbook literature. The following topics are particularly emphasised: • existence, uniqueness and continuation of solutions, • continuous dependence on initial data, • flows, • qualitative behaviour of solutions, • limit sets, • stability theory, • invariance principles, • introductory control theory, • feedback and stabilization. The last two items cover classical control theoretic material such as linear control theory and absolute stability of nonlinear feedback systems. It also includes an introduction to the more recent concept of input-to-state stability. Only a basic grounding in linear algebra and analysis is assumed. Ordinary Differential Equations will be suitable for final year undergraduate students of mathematics and appropriate for beginning postgraduates in mathematics and in mathematically oriented engineering and science.
A major portion of this book discusses work which has appeared since the publication of the book Similarity Methods for Differential Equations, Springer-Verlag, 1974, by the first author and J.D. Cole. The present book also includes a thorough and comprehensive treatment of Lie groups of tranformations and their various uses for solving ordinary and partial differential equations. No knowledge of group theory is assumed. Emphasis is placed on explicit computational algorithms to discover symmetries admitted by differential equations and to construct solutions resulting from symmetries. This book should be particularly suitable for physicists, applied mathematicians, and engineers. Almost all of the examples are taken from physical and engineering problems including those concerned with heat conduction, wave propagation, and fluid flows. A preliminary version was used as lecture notes for a two-semester course taught by the first author at the University of British Columbia in 1987-88 to graduate and senior undergraduate students in applied mathematics and physics. Chapters 1 to 4 encompass basic material. More specialized topics are covered in Chapters 5 to 7.